MODELLING SUB-NEPTUNE MASS PLANETS INTERIOR

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Introduction

The work for this lab was done using the MESA code [1]. The first step was to install the MESA code locally in the computers and after that follow the instructions in order to simulate low mass planets' interiors and evolution. The low mass gas planets that we simulate are made mostly by H and He with a rocky core.

In this report we will start by explaining how we simulated those planets, step by step. We will then look at the results and try to explain them based on previous knowledge about planetary composition and evolution.

Methods

The way we work with the code starts by creating a single planet, which we then modify in order to give it the parameters we desire. In the first step we create a single planet with $M = 30M_{earth}$. This is our initial planet and it is coreless and also has no heavy elements, which means it only consists of a mix of H and He gas.

In the next step we create a core for the planet. The core will have the same composition as that of the Earth which means that we can use the equation from [2]:

$$\frac{R}{R_\oplus} = \left(\frac{M}{M_\oplus}\right)^{0.27} \Rightarrow \rho = \rho_\oplus \left(\frac{M}{M_\oplus}\right)^{0.19} \tag{1}$$

We want to create 5 different planets with different mass cores. We can see all the different values with the calculated density in the Table 1.

Core Mass	density (gr/cm^3)
$3M_\oplus$	6.801
$5M_\oplus$	7.494
$7M_\oplus$	7.989
$10M_\oplus$	8.549
$12M_\oplus$	8.851

Table 1: The core mass of the different planets and the mean density of the core.

The next step is to reduce the mass of our planets from the $30M_\oplus$ that they are now. In order to do that we reduce the mass of the gaseous envelope, that surrounds the core. For each planet we will create 2 different planets for different values of the ratio between the mass of the envelope and the total mass of the planet.

$$f_{env} = \frac{M_{env}}{M_p} \text{ where } M_p = M_{env} + M_{core} \tag{2}$$

which leads to:

$$M_p = \frac{M_{core}}{1 - f_{env}} \tag{3}$$

After that we can create 2 planets for each core mass, by using 2 different values for the f_{env}. The final masses are shown in the Table 2:

After that we want to add an artificial luminosity that will deposit some energy inside the planets. The result of this process is the inflation of the planets which will affect the initial entropy at the base of the gaseous envelope of each planet. The artificial luminosity that we implement is $L_{center} = 2 \cdot 10^{27} erg/sec$.

Core Mass	f_{env}	Total Mass
$3M_{\oplus}$	0.1	$1.001 \cdot 10^{-5} M_{\odot}$
	0.01	$9.104 \cdot 10^{-6} M_{\odot}$
$5M_{\oplus}$	0.1	$1.669 \cdot 10^{-5} M_{\odot}$
	0.01	$1.517 \cdot 10^{-5} M_{\odot}$
$7M_{\oplus}$	0.1	$2.337 \cdot 10^{-5} M_{\odot}$
	0.01	$2.124 \cdot 10^{-5} M_{\odot}$
$10M_{\oplus}$	0.1	$3.338 \cdot 10^{-5} M_{\odot}$
	0.01	$3.035 \cdot 10^{-5} M_{\odot}$
$12M_{\oplus}$	0.1	$4.006 \cdot 10^{-5} M_{\odot}$
	0.01	$3.642 \cdot 10^{-5} M_{\odot}$

Table 2: The total mass of the planet for the different values of the core mass and f_{env}. Note that the final mass is in solar masses while the core mass is in units of earth masses.

The final step is to see how the planets evolve. During the evolution we set a new more realistic luminosity:

$$L_{center} = 5M_{core} \cdot 10^{-8} erg/sec \tag{4}$$

where M_{core} is in units of gr

We can see the results in Table 3

Core Mass	Luminosity (erg/sec)
$3M_{\oplus}$	$8.965 \cdot 10^{20}$
$5M_{\oplus}$	$1.494 \cdot 10^{21}$
$7M_{\oplus}$	$2.092 \cdot 10^{21}$
$10M_{\oplus}$	$2.988 \cdot 10^{21}$
$12M_{\oplus}$	$3.586 \cdot 10^{21}$

Table 3: The core mass of the different planets and the artificial core luminosity. Note that we only have 5 cases for the different core masses because the pairs of the planets that share the same core mass will have the same artificial luminosity added.

We then let the system evolve for a timescale of $5 \cdot 10^9 yr$ and we get our final results.

Results

The first thing we want to discuss is the evolution of the radius of the planets in a timescale of 5 billion years. There are many different literature sources that deal with the radius evolution of planets such as [3], [4]. The results of the data analysis from the MESA simulations is shown in Figure 1 and are in agreement with our previous knowledge. Here we can see the starting points and the evolution of the planetary radii. The timescale is logarithmic in order to allow us to analyze better the early evolutionary stages of the planets, because as we see in the scale of $10^9 yr$ the planets have already reached their "desired" radius and the changes are extremely small. Especially the low mass planets don't evolve after the first $10^7 yr$.

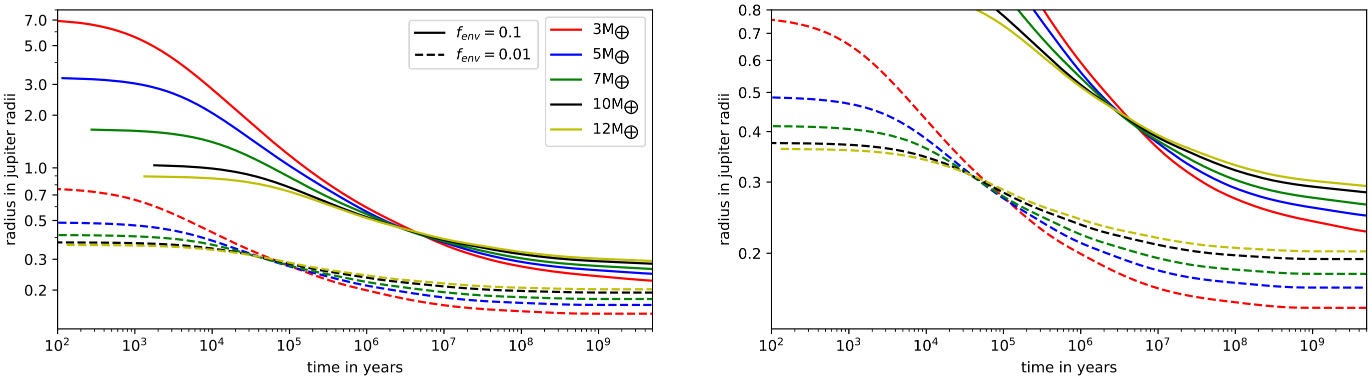

Figure 1: Time evolution of the planetary radii. The second panel focuses more on the smaller radii.

The heavier planets evolve up until the first Gyr where they almost reach their final radius.

The next thing one notices is that the planets with small mass cores have bigger initial radii than the ones with higher mass cores. This has to do with the way the planets were created using the MESA simulation. What we also see is that this thing only lasts for a short period of time. Around $4 \cdot 10^6 yr$ for the $f_{env} = 0.1$ planets and $8 \cdot 10^4 yr$ for the $f_{env} = 0.01$ planets, the order of the planetary radii reverses. The planets keep evolving and their final radii is in the scale of a magnitude of order smaller than their starting radii, with the low mass core planets having the biggest change.

We can also see 2 distinct categories in this diagram. The planets that are characterized by $f_{env} = 0.1$ are clearly distinct from the $f_{env} = 0.01$ planets. Their starting radii is much different (almost an order of magnitude) and they have specific turning points (where the order of radii reverses). We also see that the difference between their final radii is much smaller than their starting, which becomes clear by using the second panel of the Figure 1.

M_{core}	f_{env}	mass	radius	M_{core}	f_{env}	mass	radius
$3M_\oplus$	0.01	$0.0095\ M_j$	$0.147R_j$	$7M_\oplus$	0.1	$0.0245\ M_j$	$0.263\ R_j$
$3M_\oplus$	0.1	$0.0104\ M_j$	$0.226\ R_j$	$10M_\oplus$	0.01	$0.0318\ M_j$	$0.194\ R_j$
$5M_\oplus$	0.01	$0.0159M_j$	$0.164R_j$	$10M_\oplus$	0.1	$0.0350\ M_j$	$0.283\ R_j$
$5M_\oplus$	0.1	$0.0175\ M_j$	$0.248R_j$	$12M_\oplus$	0.01	$0.0382\ M_j$	$0.202\ R_j$
$7M_\oplus$	0.01	$0.0223\ M_j$	$0.178R_j$	$12M_\oplus$	0.1	$0.0420\ M_j$	$0.293\ R_j$

Table 4: The final values of the mass and radius for all the simulated planets

After that we want to compare the final mass and radius of the planets we simulated with the ones detected. In Table 4 we give these values for all the different planets that we created .We use [5] to get a list of all the exoplanet detections. We only focus on the ones that we have a specific mass and radius measurement. In Figure 2 we have 2 panels. In the right panel we can see the comparison of our simulated planets with the vast majority of exoplanet detections. We see that our planets fall in low mass area that is populated by many different exoplanets.

In the left panel of Figure 2 we have a more zoomed figure in the area that our exoplanets exist. We have also marked 10 different exoplanets that have measured similar mass and radius with the simulated ones.

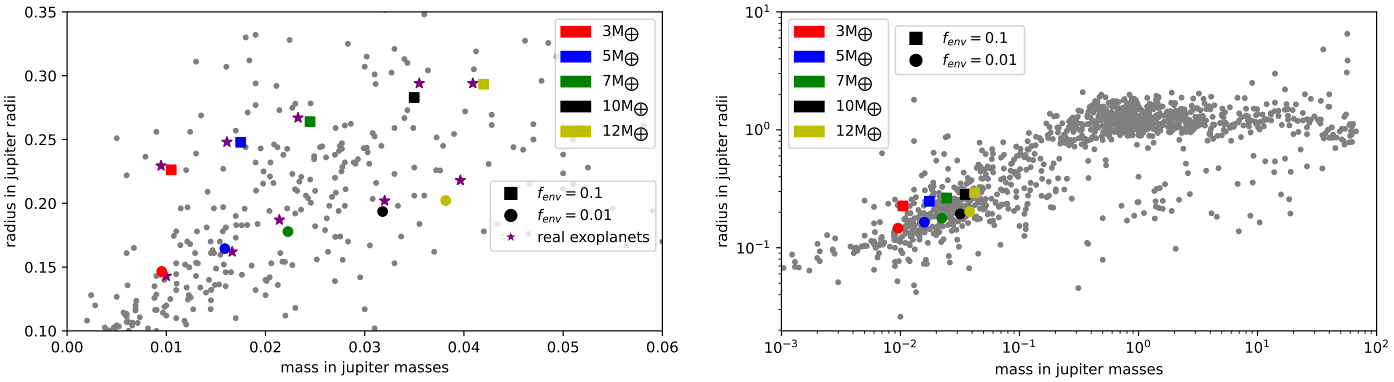

Figure 2: The final mass and radius of the simulated planets. The grey dots are all the exoplanets with well measured mass and radius. For a more clear overview use the right panel. There is also an indication of some cases of exoplanets that can also be seen in Table 5 and have similar parameter values with the simulated planets.

The 10 real exoplanets were chosen just by the similarity of the masses and radii. In Table 5 we have noted their parameters and what type of planets they most likely are based on the catalog in [6]. As one can see, in most cases we have Neptune-like planets, which is something we expect in that range of masses and radii. We also expect that since our planets also belong in this category. We also have some Super Earths, which we also expect since these two categories have similar masses and mainly differ in the composition.

mass	radius	name	type	mass	radius	name	type
0.0100	0.143	K2-133 c	Super Earth	0.0233	0.267	Kepler-223 b	Neptune-like
0.0095	0.229	TOI-178 d	Neptune-like	0.0320	0.202	TOI-1062 b	Super Earth
0.0166	0.162	K2-111 b	Super Earth	0.0355	0.294	K2-266 b	Neptune-like
0.0161	0.248	Kepler-26 b	Neptune-like	0.0396	0.218	HD 106315 b	Super Earth
0.0214	0.187	K2-286 b	Neptune-like	0.0409	0.294	K2-138 e	Neptune-like

Table 5: Information about the real exoplanets shown in Figure 2, with similar mass and radius with the simulated planets.

The next thing we want to discuss is the way that energy is transported in the gaseous envelopes of these planets. We can see the comparison between the radiative and adiabatic gradient for all the different cases in Figure 3. We see that all the planets have similar profiles so our analysis can be the same for all of them.

Both gradients are almost equal at the smaller distances from the core, with the radiative gradient being a little bit more dominant than the adiabatic. As we go more into the outer regions of the envelope we that the radiative gradient has much greater values than the adiabatic. This leads to the conclucion that the main way of heat transfer [7], especially in the outer regions is convection. With the radiative gradient rapidly dwindling in the very edges of the planet, we have the adiabatic gradient becoming bigger than the radiative for a small part of the planet. We should expect that the radiation is the main way heat transfers in that region.

The final result of this analysis is the Temperature vs Pressure profiles of the simulated planets. We find similar profiles to what we expect from various literature sources such as [8],[9]. In Figure 4 we plot these profiles for all our cases. We can observe that the profiles are very similar, which is something we should have expected.

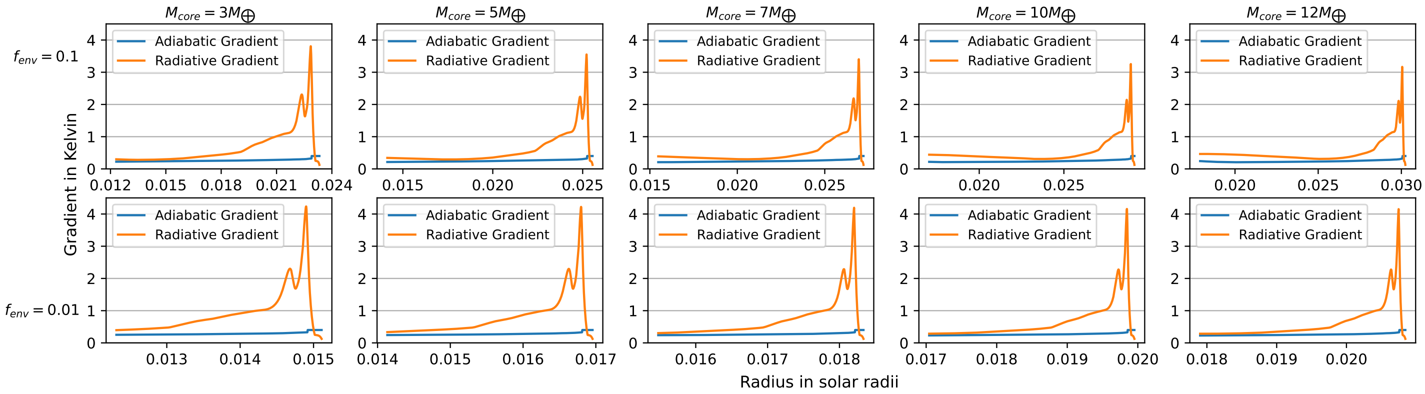

Figure 3: The radiative and adiabatic gradient as a function of the radius for all the different simulated planets. For the different panels, from left to right the core mass increases. The top panels are for $f_{env} = 0.1$ and the bottom panels for $f_{env} = 0.01$.

The reason for that is that all the planets have similar masses and composition which should lead to similar profiles with the differences being mainly the extreme values of each case. As we can see the maximum temperature ranges between $\approx 1000K$ and $> 2500K$, while the maximum pressure ranges from $\approx 1.5 \cdot 10^{10} \frac{dyn}{cm^2}$ to $\approx 9 \cdot 10^{10} \frac{dyn}{cm^2}$. Although the extreme values can differ the shape of the profiles of all the planets remains the same.

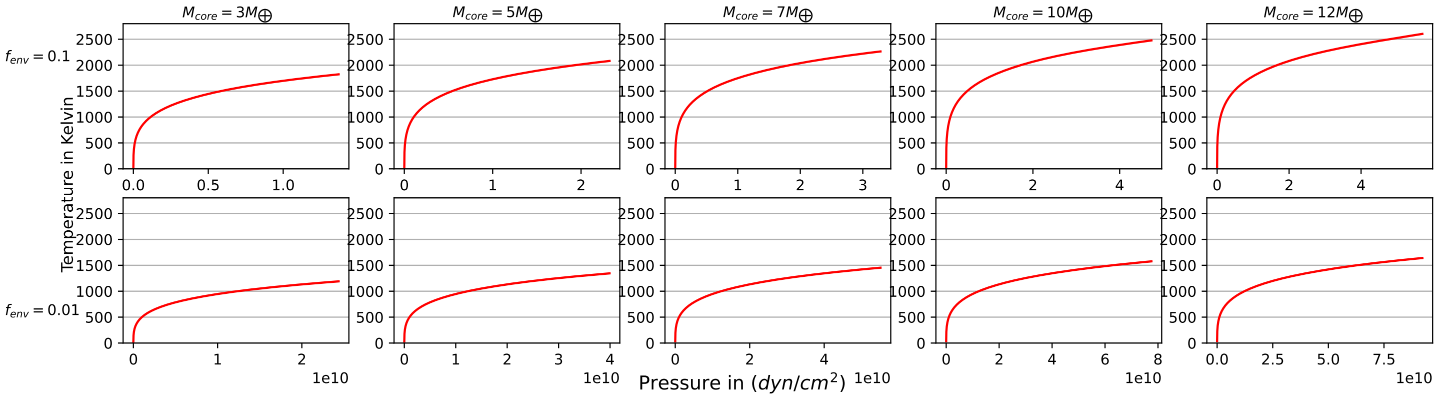

Figure 4: The Temperature vs Pressure profiles for all the simulated planets. From left to right the core mass increases. The top panels are for $f_{env} = 0.1$ and the bottom panels for $f_{env} = 0.01$.

Conclusion

We have used the MESA code in order to simulate 10 different planets with sub-neptune masses. From the analysis of the data from the simulation we were able to gain a useful insight on how these planets evolve. We also gained information on the ways they transport heat and their temperature-pressure profiles for all the different cases.

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